BC Calculus Quiz #12 v.A NC
More Integration Dr. Wiz Spring 2020

18pts

Name Solution Baibafsü Ei **Squrtle**

Instructions: Solve each of the problems below. Please show your work (for partial credit) and box or circle your answers. **A calculator is not permitted on this portion of the quiz.**

1. (9 Pts) Evaluate each of the following integrals using the given technique.

3pts a. $\int_0^1 x^2 e^{-2x^3}\,dx$ (u-substitution) Let $u=-2x^3$, $du=-6x^2dx$ when $x=0$, $u=0$; $x=1$, $u=-2$

$\int_0^1 x^2 e^{-2x^3}dx = -\frac{1}{6}\int_0^1 -6x^2 e^{-2x^3}dx$

$= -\frac{1}{6}\int_0^{-2} e^u du = \frac{1}{6}\int_{-2}^{0} e^u du = \frac{1}{6}e^u\Big|_{-2}^{0} = \frac{1}{6}(e^0-e^{-2}) = \boxed{\frac{1}{6}(1-e^{-2})}$

$\approx .144$ ✓ FINFNT

2pts b. $\int \tan 2x\,dx$ (log rule) $= \int \frac{\sin 2x}{\cos 2x}dx = -\frac{1}{2}\int \frac{-2\sin 2x}{\cos 2x}dx$

$= \boxed{-\frac{1}{2}\ln|\cos 2x| + C}$

−1 for correct answer w/o log rule
−.5 for no $\frac{1}{2}$ in front

2pts c. $\int \frac{2}{x^2-6x+13}\,dx$ (complete the square)

$x^2-6x+9+13-9$
$(x-3)^2+4$

$\int \frac{2}{(x-3)^2+4}dx$ Let $u=x-3$, $du=dx$

$\int \frac{2}{u^2+2^2}du = 2\int\frac{1}{u^2+2^2}du = 2\cdot\frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right)+C$

$\tan^{-1}\left(\frac{u}{2}\right)+C = \boxed{\tan^{-1}\left(\frac{x-3}{2}\right)+C}$

2pts d. $\int \frac{x^2-10x+1}{x-7}\,dx$ (Long division)

$$\begin{array}{r} x-3 \\ x-7\;\big)\; x^2-10x+1 \\ -(x^2-7x) \\ \hline -3x+1 \\ -(-3x+21) \\ \hline -20 \end{array}$$

$\int \frac{x^2-10x+1}{x-7}dx = \int\left(x-3-\frac{20}{x-7}\right)dx$

$= \boxed{\frac{x^2}{2}-3x-20\ln|x-7|+C}$

2. (9 Pts) Evaluate each of the following integrals.

a. $\int \frac{1-2x}{\sqrt{16-x^2}}\,dx = \int \frac{1}{\sqrt{16-x^2}}\,dx + \int \frac{-2x}{\sqrt{16-x^2}}\,dx$ (I) (II)

(I) $\int \frac{1}{\sqrt{16-x^2}}\,dx = \sin^{-1}\left(\frac{x}{4}\right) + C$

(II) Let $u = 16-x^2$
$du = -2x\,dx$

$\int \frac{-2x}{\sqrt{16-x^2}}\,dx = \int \frac{du}{u^{1/2}} = \int u^{-1/2}\,du = 2u^{1/2} + C$
$= 2\sqrt{16-x^2} + C$

$\therefore \int \frac{1-2x}{\sqrt{16-x^2}}\,dx = \boxed{\sin^{-1}\left(\frac{x}{4}\right) + 2\sqrt{16-x^2} + C}$

b. $\int \frac{3x^2+x}{2x^3+x^2+4}\,dx = \frac{1}{2}\int \frac{2(3x^2+x)}{2x^3+x^2+4}\,dx = \frac{1}{2}\int \frac{6x^2+2x}{2x^3+x^2+4}\,dx$

$= \boxed{\frac{1}{2}\ln\left|2x^3+x^2+4\right| + C}$

$\cos \pi/3 = 1/2$
$\sec \pi/3 = 2$

Let $u = \sec x$
$du = \sec x \tan x\,dx$
when $x=0,\ u=1$
$x=\pi/3,\ u=2$

c. $\int_0^{\pi/3} \sec^3 x\ \tan x\,dx$

$\int_1^2 u^2\,du = \left.\frac{u^3}{3}\right|_1^2 = \frac{2^3}{3} - \frac{1^3}{3} = \frac{8}{3} - \frac{1}{3} = \boxed{7/3}$

✓ FNINT

Let $u = 4x^2-1$
$du = 8x\,dx$

d. $\int x(4x^2-1)^{3/2}\,dx$

$= \frac{1}{8}\int 8x(4x^2-1)^{3/2}\,dx = \frac{1}{8}\int u^{3/2}\,du = \frac{1}{8}\cdot\frac{2}{5}u^{5/2} + C$

$= \frac{1}{20}u^{5/2} + C = \boxed{\frac{1}{20}(4x^2-1)^{5/2} + C}$